Quantum kinetic theory IV: Intensity and amplitude fluctuations of a Bose-Einstein condensate at finite temperature including trap loss

D. Jaksch[1], C.W. Gardiner[2], K.M. Gheri[1] and P. Zoller[1]
[1] *Institut für Theoretische Physik, Universität Innsbruck, 6020 Innsbruck, Austria*
[2] *Physics Department, Victoria University, Wellington, New Zealand*

We use the quantum kinetic theory to calculate the steady state and the fluctuations of a trapped Bose-Einstein condensate at a finite temperature. The system is divided in a condensate and a noncondensate part. A quantum-mechanical description based on the number conserving Bogoliubov method is used for describing the condensate part. The noncondensed particles are treated as a classical gas in thermal equilibrium with temperature T and chemical potential μ. We find a master equation for the reduced density operator of the Bose-Einstein condensate, calculate the steady state of the system, and investigate the effect of one-, two-, and three-particle losses on the condensate. Using linearized Ito equations we find expressions for the intensity fluctuations and the amplitude fluctuations in the condensate. A Lorentzian line shape is found for the intensity correlation function that is characterized by a time constant γ_I^{-1} derived in the paper. For the amplitude correlation function we find ballistic behavior for time differences smaller than γ_I^{-1}, and diffusive behavior for larger time differences.

I. INTRODUCTION

In a series of papers we have developed a quantum kinetic (QK) theory with application to Bose condensation of cold dilute gases. In the first two papers, which we shall refer to as QKI [1] and QKII [2], we considered a spatially homogeneous, weakly condensed system, where the interaction between the atoms was assumed to be sufficiently weak for quasiparticle effects to be negligible. In QKIII [3] the theory was extended to a strongly condensed gas in a trapping potential under the assumption that the noncondensed vapor acts as a heat and particle reservoir for the condensate (see also Ref. [4]), a situation which corresponds closely to present experiments of Bose condensation with alkali-metal vapors [5–11].

In the present paper (QKIV) we will study the steady state, amplitude, and phase fluctuations of a trapped Bose-Einstein condensate at *finite temperature*, including the effects of one-, two-, and three particle losses on the condensate. Such a study seems particularly timely in view of the present interest in the dynamics and measurement of the phase of the Bose condensate (for a review see Ref. [12]). Until now the discussion in the literature has essentially focused on phase collapse or diffusion, and phase revivals in the zero-temperature limit, analyzing the dependence of collapse and revival times on the trap potential, the dimensionality of the gas, atom number fluctuations, and the coherent dynamics of the condensate [13–15]. In contrast, in the present work we will study in detail fluctuations as a result of interaction of the condensate with a (reservoir of) uncondensed atoms.

We will almost exclusively consider a grand canonical particle reservoir in this work. This particle reservoir will be assumed to have a constant chemical potential and not to be influenced by the mean field of the condensate. The results will therefore only be valid in the case of small condensate and large thermal particle numbers. In all other cases the simple model used here for the thermal particles will have to be replaced by a more sophisticated one. A more detailed discussion on the expected correction to the results due to particle conservation and mean-field effects is given in Appendix A.

The starting point of the theory is to decompose the field operator describing the N-particle system into condensate and noncondensate parts. The division is based on a splitting into two energy regions where the high energy band is supposed to contain particles in a thermal state corresponding to a given temperature T, and chemical potential μ. The condensate band contains the actual condensate as well as quasi-particle excitations. A quantum-mechanical description based on the number conserving Bogoliubov method is used for describing the condensate part [16]. To facilitate the analysis we drop the quasi-particles and describe the condensate by a single mode with destruction operator B, and the spatial wave function $\xi_N(\mathbf{x})$, corresponding to the solution of the Gross-Pitaevskii equation for N condensate particles. Elimination of the noncondensate part leaves us with a master equation for the (reduced) condensate density operator. The physics contained in this equation is quite rich. The master equation accounts for loss/gain of particles to/from the thermal band, and phase-destroying but number-conserving collisions between condensate and noncondensate particles as well as linear and nonlinear trap loss.

A diagrammatic illustration of the processes described by the master equation is given in Fig. 1 . We realize that there are two types of processes. On the one hand, there are those which involve particles from both the condensate and the noncondensed band. They comprise processes which lead to a redistribution of particles between the two bands at rates $W^+(N)$ (condensate gain) and $W^-(N)$ (condensate loss), as well as number conserving scattering events of thermal particles off the condensate. The latter occur with a rate $R_{00}(N)$ and will give rise to fluctuations in the condensate phase. Explicit expres-

sions for $W^{\pm}$ and R_{00} will be given below. On the other hand there are several loss mechanisms at work which deplete the condensate [17,18]. There is one-particle loss due to collisions with background gas atoms with associated rate γ_1. Two particles can be lost with rate $\gamma_2(N)$ from the condensate if two condensed particles collide, and one of them changes its internal state. This particle no longer sees the trap, and escapes. Its partner is imparted with the energy difference set free by the collision, and is also lost from the trap. Finally, three-particle loss can occur with rate $\gamma_3(N)$ if in a three-particle collision a dimer is formed. The binding energy is imparted to the third particle, and all of them escape from the trap. Note that the description of the noncondensate particles in terms of a thermal reservoir results in a finite occupation of the condensate mode even in the presence of loss channels.

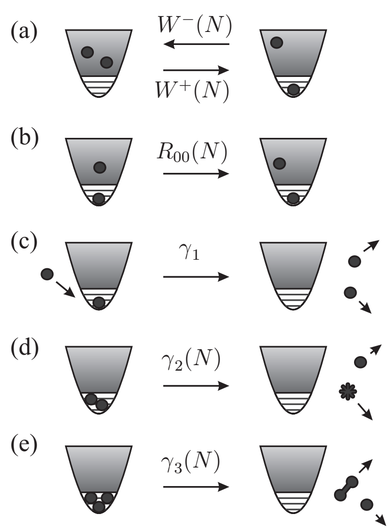

Fig. 1. Interpretation of the processes described by the master equation: (a) $W^+(N)$ and $W^-(N)$ are the feeding and depletion rates of the condensate from and to the noncondensed thermal band of particles, respectively. (b) R_{00} is the rate of thermal particles bouncing off the condensate without changing its occupation number. γ_1 (c), $\gamma_2(N)$ (d) and $\gamma_3(N)$ (e) are the rates of one,- two,- and three-particle losses, respectively. We use a star to indicate a change in the internal state of a particle. The barbell represents two atoms having formed a molecule.

II. MAIN RESULTS

In this section we will give a short overview of the main results to be derived in detail in the remainder of this paper. As a starting point for our analysis of the particle number and phase fluctuations of a Bose-Einstein condensate at a finite temperature, we adopt the theoretical description developed in the precursor papers QKI-III [1–3].

A. Fluctuation analysis

In the limit of large condensate particle numbers, we may approximate the master equation [Eq. (17)] by an equation that is of Lindblad type. This has the advantage that standard techniques developed in quantum optics for the description of fluctuation properties become applicable [19]. We have thus derived quantum stochastic differential equations for the condensate particle number $\tilde{N} = B^\dagger B$ and the Glogower-Susskind phase operator $e^{i\phi}$, which is known to characterize phase fluctuations well in the limit of large occupation numbers [20]. Linearization of these equations is permissible in the very same limit of large average occupation $\bar{N} = \langle \tilde{N} \rangle$, and allows us to work out the two-time correlation functions of occupation number and phase. The spectra of condensate occupation number and amplitude fluctuations are then immediately obtainable by Fourier transformation.

1. Condensate particle number fluctuations

For the correlation function of the particle number fluctuations [21] we obtain the following result which holds in the stationary limit, i.e., for times satisfying $t + s \gg \gamma_I^{-1}$

$$\langle \tilde{N}(t), \tilde{N}(s) \rangle = \frac{\bar{f}_2}{2\gamma_I} \mathrm{e}^{-\gamma_I |t-s|} = \sigma^2 \mathrm{e}^{-\gamma_I |t-s|}, \qquad (1)$$

with $\langle a, b \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle$. In Eq. (1)

$$\bar{f}_2/2 = \bar{N}(\bar{W}^+ + \bar{W}^- + \gamma_1) + 4\bar{N}^2 \bar{\gamma}_2 + 9\bar{N}^3 \bar{\gamma}_3, \qquad (2)$$

with a bar denoting evaluation at $N = \bar{N}$. Note that σ appearing in Eq. (1) is a measure of the width of the particle number distribution. The characteristic time constant γ_I with which the particle number fluctuations regress is given by

$$\gamma_I = 2\bar{N} \partial_{\bar{N}} \bar{W}^- - (\bar{\gamma}_2 + 3\bar{\gamma}_3 \bar{N}) 8\bar{N}/5. \qquad (3)$$

$\partial_{\bar{N}}$ denotes the derivative with respect to N and evaluation at $N = \bar{N}$. The exact size of this rate depends on the specific experimental setup. A convenient quantity to assess the amount of fluctuations present is the well known Mandel Q parameter defined as

$$Q = \lim_{t \to \infty} \langle \tilde{N}(t), \tilde{N}(t) \rangle / \langle \tilde{N}(t) \rangle - 1. \qquad (4)$$

A coherent state would correspond to a value of $Q = 0$, while a number state yields the minimum value $Q = -1$. Assuming two and three particle losses to be insignificant we find for the Mandel Q parameter,

$$Q \approx \frac{5kT}{2\mu_{\bar{N}}} - 1. \qquad (5)$$

To arrive at this result the approximate expressions of the rates W^+ and W^- as given in Eqs. (19) in terms of the scattering length a, the reservoir temperature T and chemical potential μ have been used.

The results (for details, see Sec. IV C) can now be summarized as follows. The variance of the occupation of the condensate mode is proportional to $kT/\mu_{\bar{N}}$, with $\mu_{\bar{N}}$ the chemical potential for the condensate mode with average occupation $\bar{N}$. In the Thomas-Fermi approximation μ_N is given by

$$\mu_N = \left(\frac{15Num^{3/2}\omega_x\omega_y\omega_z}{16\sqrt{2}\pi}\right)^{2/5}. \tag{6}$$

The constants used in Eq. (6) are defined in Sec. III A. The characteristic rate at which the correlation function drops off is roughly given by $\gamma_I \approx 2\bar{N}\partial_{\bar{N}}\bar{W}^-$, with W^- the loss rate to the noncondensed band.

2. Amplitude and phase fluctuations

In the limit of large and well-defined average occupation number $\bar{N}$ of the condensatemmode, the amplitude correlation function $\langle B^\dagger(t)B(s)\rangle$ is well suited to assess phase properties of the condensate mode [21]. In particular, the spectrum of phase fluctuations is identical to the spectrum of amplitude fluctuations. For the amplitude correlation function we obtain for $t > s$

$$\langle B^\dagger(t)B(s)\rangle = \bar{N}\mathrm{e}^{\left(i\frac{\mu_{\bar{N}}}{\hbar} - \frac{16}{25}\bar{R}_{00}\right)(t-s) - \eta\left(\gamma_I(t-s) + \mathrm{e}^{-\gamma_I(t-s)} - 1\right)}, \tag{7}$$

where $\eta = \left(\sigma\partial_{\bar{N}}\mu_{\bar{N}}/\gamma_I\hbar\right)^2$. As we will see in Sec. II A 3 R_{00} is negligible in the above correlation function [Eq. (7)] for most of the current experiments. The structure of the correlation function indicates that there are two distinct time regimes:

- For $\gamma_I|t-s| \ll 1$, the term proportional to η in the exponent is proportional to $(t-s)^2$. This is called the ballistic regime.
- For $|t-s|\gamma_I \geq 1$, the phase behaves like that of a system undergoing phase diffusion. A characteristic of such behavior is a linear dependence of the exponent on $|t-s|$. Note that for large time differences we observe the legacy of the ballistic regime in the form of a rescaling of $\bar{N}$ to $\bar{N}_\infty = \bar{N}e^\eta$.

3. Numerical values

Using data from the experiments at JILA [22] and an average occupation number of $\bar{N} = 25000$ Rubidium atoms at a temperature $T = 0.5\mu K$ in a trap with $f_x = f_y = f_z/\sqrt{8} = 47$Hz we obtain for the rates $\gamma_I \approx 2$Hz, $\bar{R}_{00} \approx 4$mHz, $\bar{W}^\pm \approx 50$Hz, and $\eta \approx 800$. These values have to be understood as order of magnitude estimates for current experiments. Note that we used a value of $a = 2.6$nm for the scattering letngth of rubidium whereas recent experiments at JILA showed that $a = 5.1$nm.

III. MODEL

In this section we briefly describe the basic concepts of the quantum kinetic theory developed in Ref. [1–3,16,23]. We do not give a detailed derivation of the master equation used throughout the paper since this can be found in Ref. [3].

A. Description of the system

The Hamiltonian of a weakly interacting Bose gas confined by a potential $V_T(\mathbf{x})$ in second quantization is written as

$$\begin{aligned} H = &\int d^3\mathbf{x}\,\psi^\dagger(\mathbf{x})\left(-\frac{\hbar^2}{2m}\nabla^2 + V_T(\mathbf{x})\right)\psi(\mathbf{x}) \\ &+\frac{1}{2}\int d^3\mathbf{x}\int d^3\mathbf{x}'\,\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')u(\mathbf{x}-\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}). \end{aligned} \tag{8}$$

$\psi(\mathbf{x})$ is the standard bosonic field operator. The two-body interaction potential $u(\mathbf{x}-\mathbf{x}')$ is a short-range potential of the form $u\delta(\mathbf{x}-\mathbf{x}')$ where $u = 4\pi\hbar a/m$ with a the scattering length. We assume the trapping potential to be of the form $V_T(\mathbf{x}) = m(\omega_x^2x^2 + \omega_x^2y^2 + \omega_z^2z^2)/2$.

As was shown in Ref. [3] we can obtain a master equation for the density operator of the condensate mode by dividing the Bose gas into two energy regions called the condensate band R_C and the noncondensate band R_{NC}. The boundary E_R between these two regions is chosen according to Ref. [3] such that R_{NC} is not significantly affected by the mean field of the condensate. The field operator is then written as $\psi(\mathbf{x}) = \psi_{\mathrm{NC}}(\mathbf{x}) + \phi(\mathbf{x})$ describing particles in the noncondensate band and in the condensate band, respectively. We will treat the particles in R_{NC} as classical thermalized particles characterized by a temperature T and a chemical potential μ. The particles in R_C are affected by the mean field, and they have to be treated quantum-mechanically. R_C contains all the trap levels that are significantly modified by the mean field of the condensate. As in Ref. [23] we will use the simplest possible description of the condensate band by assuming that only one mode, namely, the condensate mode itself, is important and all the other modes are negligible. The master equation we will use for our calculations is an equation for the reduced density operator ρ_c of the condensate band R_C interacting with the bath of particles in R_{NC}.

1. Condensate band R_C

We use the number-conserving Bogoliubov method derived in Ref. [16] to describe the particles in R_C. In this formulation we can write down the condensate band field operator $\phi(\mathbf{x})$ in the form

$$\phi(\mathbf{x}) = B(N)\left\{\xi_N(\mathbf{x}) + \sum_m \frac{b_m f_m(\mathbf{x}) + b_m^\dagger g_m(\mathbf{x})}{\sqrt{N}}\right\}. \quad (9)$$

The annihilation operator $B(N-1)$ brings the system R_C from the ground state with N atoms to the ground state with $N-1$ atoms. The action of the operator $B(N-1)$ to the condensate is depicted in Fig. 2 . $|N\rangle_N$ denotes the ground state with N particles in the condensate. Applying the operator B to this state yields $B|N\rangle_N = \sqrt{N}|N-1\rangle_{N-1}$. Note that the operator B changes the chemical potential of the condensate from μ_N to μ_{N-1}.

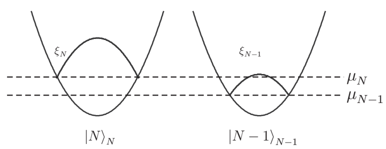

Fig. 2. Condensate ground state with N particles $|N\rangle_N$ and with $N-1$ particles $|N-1\rangle_{N-1}$. These two states are connected by the operator B, as described in the text.

As shown in Ref. [16] $\xi_N(\mathbf{x})$ is the condensate wave function, and satisfies the Gross-Pitaevskii equation

$$-\frac{\hbar^2}{2m}\nabla^2\xi_N + V_T\xi_N + Nu\left|\xi_N\right|^2\xi_N = \mu_N\xi_N. \quad (10)$$

In all our calculations we will use the Thomas-Fermi approximation for the chemical potential [Eq. (6)] and for the condensate wave function

$$\xi_N(\mathbf{x}) = \sqrt{\frac{\mu_N - V_T(\mathbf{x})}{Nu}} \quad \text{for } \mu_N > V_T(\mathbf{x}), \quad (11)$$

and zero elsewhere. The energy of the condensate is given by

$$E_0 = \frac{5N\mu_N}{7} \quad (12)$$

The amplitudes $f_m(\mathbf{x})$ and $g_m(\mathbf{x})$ describe creation and destruction of quasiparticles. They are defined in Ref. [3], but we will not need them in this paper. We will assume there is always a fairly large mean number of particles $\bar{N}$ in the condensate so that we can neglect the influence of the quasiparticles on $\phi(\mathbf{x})$ [24].

2. Noncondensate band R_{NC}

We treat the noncondensate band as a thermal bath of particles in thermal equilibrium. According to Ref. [3] we only need the phase-space density of the noncondensed particles $F(\mathbf{K},\mathbf{x})$ to calculate all the rates appearing in the master equation for ρ_c [Eq. (17)]. In our calculations we will use the classical approximation

$$F(\mathbf{K},\mathbf{x}) = \mathrm{e}^{(\mu - \frac{\hbar^2\mathbf{K}^2}{2m} - V_T(\mathbf{x}))/kT}. \quad (13)$$

We expect corrections to the rates $W^\pm$ and R_{00} from using a more detailed description of the noncondensed particles. However, all the calculations presented here will remain valid, since they are mostly independent of the functional form of the rates $W^\pm$ and R_{00}. We only require that it is permissible to linearize the rates $W^\pm(N)$ and $R_{00}(N)$ around the mean number of particles in the condensate $\bar{N}$. In the present work we will mainly present results obtained by using Eqs. (19) and (20) for the rates $W^\pm(N)$ and $R_{00}(N)$ which were obtained by the assumption of a grand canonical classical bath of particles not influenced by mean-field effects. Corrections to the stationary state due to conservation of the total number of particles as well as mean-field effects and quantum statistics effects are estimated in Appendix A. A further, more detailed, discussion of different models for the uncondensed bath (i.e., the noncondensate band) lies beyond the scope of this paper, and will be presented in other work. However, we want to point out that these models might include canonical baths with a constant particle number [25], evaporatively cooled baths [26] as well as baths that are continuously fed with particles [27].

3. Trap loss

There are several processes leading to losses of condensate particles from the trap. We will consider one,- two,- and three-particle loss with loss rates γ_1, $\gamma_2(N)$ and $\gamma_3(N)$, respectively. One-particle loss might be caused by background gas particles hitting condensate particles or by coupling condensate particles out of the trap as described in Ref. [21]. Inelastic two-particle collisions changing the internal properties of the particles lead to two particle loss. In most of the current experiments the two particle loss is negligible compared to the three particle loss caused by the inelastic collision of three particles. In the Thomas-Fermi approximation we obtain for the loss rates

$$\gamma_1 = \frac{K_1}{2}\int d^3x\ |\xi_N(x)|^2 = \frac{K_1}{2}, \quad (14)$$

$$\gamma_2(N) = \frac{K_2}{4}\int d^3x\ |\xi_N(x)|^4 = \frac{K_2 16\cdot\pi\mu_N^{7/2}\sqrt{2}}{105m^{3/2}\omega_x\omega_y\omega_z N^2u^2} \propto N^{-3/5}, \quad (15)$$

$$\gamma_3(N) = \frac{K_3}{6}\int d^3x\,|\xi_N(x)|^6 = \frac{4K_3\mu_N\gamma_2(N)}{9K_2Nu} \propto N^{-6/5}. \tag{16}$$

For Rubidium the constants K_i have been measured in [17]. The rates K_i have been calculated analytically in Refs. [28–30].

B. Master equation

We simplify Eq. (50) in Ref. [3] for the case that the condensate band consists only of the condensate mode alone. In contrast to Ref. [3] we keep terms including R_{00}, and add the loss terms to the master equation (see Fig. 1). We thus obtain the following equation for the reduced density operator of the condensate band ρ_c:

$$\begin{aligned}
\dot{\rho}_c = &-\frac{i}{\hbar}[H_0,\rho_c] &(17a)\\
&+2B^\dagger\{W^+(\hat N)\rho_c\}B - [BB^\dagger,\{W^+(\hat N)\rho_c\}]_+ + 2B\{W^-(\hat N)\rho_c\}B^\dagger - [B^\dagger B,\{W^-(\hat N)\rho_c\}]_+ &(17b)\\
&+2BB^\dagger\{R_{00}(\hat N)\rho_c\}BB^\dagger - [BB^\dagger BB^\dagger,\{R_{00}(\hat N)\rho_c\}]_+ &(17c)\\
&+2B\{\gamma_1\rho_c\}B^\dagger - [B^\dagger B,\{\gamma_1\rho_c\}]_+ + 2BB\{\gamma_2(\hat N)\rho_c\}B^\dagger B^\dagger - \Big[B^\dagger B^\dagger BB,\{\gamma_2(\hat N)\rho_c\}\Big]_+ &(17d)\\
&+2BBB\{\gamma_3(\hat N)\rho_c\}B^\dagger B^\dagger B^\dagger - \Big[B^\dagger B^\dagger B^\dagger BBB,\{\gamma_3(\hat N)\rho_c\}\Big]_+, &(17e)
\end{aligned}$$

where $\hat N\rho_c = \left[B^\dagger B,\rho_c\right]_+/2$. An intuitive interpretation of the processes described by the master equation [Eq. (17)] is given in Fig. 1 . The free evolution of the condensate is described by the Hamiltonian H_0,

$$H_0 = E_0(\tilde N) \quad \text{where} \quad E_0(N+1) - E_0(N) = \mu_N, \tag{18}$$

and μ_N is the chemical potential obtained from the Gross-Pitaevskii equation.

Growth and depletion of the condensate due to interaction with the noncondensed particles is given in Eq. (17b). The growth and the depletion rates $W^\pm(N)$ of the condensate were already calculated in Ref. [3] using the Maxwell-Boltzmann approximation for the phase-space density of the noncondensed particles. They are given by

$$W^+(N) = \frac{4m(akT)^2}{\pi\hbar^3}\mathrm{e}^{2\mu/kT}\left\{\frac{\mu_N}{kT}K_1\Big(\frac{\mu_N}{kT}\Big)\right\}, \tag{19a}$$

$$W^-(N) = \mathrm{e}^{(\mu_N-\mu)/kT}W^+(N). \tag{19b}$$

The rate R_{00} defined in Eq. (141) of Ref. [3] can be understood as describing the process of thermal particles bouncing off the condensate. This process does not change the number of particles in the condensate, but does cause phase fluctuations. Using the above expression [Eq. (13)] for $F(\mathbf{K},\mathbf{x})$ and the Thomas-Fermi approximation for the condensate wave function, we find

$$R_{00}(N) = \frac{4kT\mu_N^4}{9\pi^4\hbar^5\omega_x^3\omega_z N^2}\mathrm{e}^{\mu/kT}\frac{\operatorname{arsinh}\left(\sqrt{\frac{\omega_z^2-\omega_x^2}{\omega_x^2}}\right)}{\sqrt{\frac{\omega_z^2-\omega_x^2}{\omega_x^2}}} \tag{20a}$$

for $\omega_z > \omega_x = \omega_y$, and

$$R_{00}(N) = \frac{4kT\mu_N^4}{9\pi^4\hbar^5\omega_x^3\omega_z N^2}\mathrm{e}^{\mu/kT}\frac{\arcsin\left(\sqrt{\frac{\omega_x^2-\omega_z^2}{\omega_x^2}}\right)}{\sqrt{\frac{\omega_x^2-\omega_z^2}{\omega_x^2}}} \tag{20b}$$

for $\omega_z < \omega_x = \omega_y$. A detailed derivation of R_{00} is given in Appendix B.

Trap loss is accounted for by the last two lines of the master equation Eqs. (17d) and (17e). Note that the only difference between the process including the rate $W^-(N)$ and the one-particle loss rate γ_1 is the dependence of the two rates on the properties μ and T of the noncondensate.

IV. SOLUTIONS OF THE MASTER EQUATION

In this section we investigate solutions of Eq. (17). We find the stationary solution and derive a differential equation for the mean number of particles in the condensate $\bar N$. Using linearized Ito equations, we obtain the correlation functions $\langle N(t), N(s)\rangle$ and $\langle B^\dagger(t)B(s)\rangle$.

We define

$$g_N^\nu := {}_N\langle N-\nu|\rho_c|N\rangle_N. \tag{21}$$

From Eq. (17), we derive the evolution equation of the matrix elements, expand the terms in this equation for large N in a Kramers-Moyal type expansion to order $1/N$, and obtain

$$\dot g_N^\nu = 2\sqrt{(N-\nu)N}\left\{W^+(N-1)g_{N-1}^\nu - W^-(N)g_N^\nu\right\}$$

$$+ 2\sqrt{(N-\nu+1)(N+1)}\left\{W^-(N+1)g^\nu_{N+1} - W^+(N)g^\nu_N\right\}$$
$$+ 2\sqrt{(N-\nu+1)(N+1)}\gamma_1 g^\nu_{N+1} - (2N-\nu)\gamma_1 g^\nu_N$$
$$+ 2\sqrt{(N-\nu+1)(N-\nu+2)(N+1)(N+2)}\gamma_2(N)g^\nu_{N+2} - ((N-\nu)(N-\nu-1)+N(N-1))\gamma_2(N)g^\nu_N$$
$$+ 2\sqrt{(N-\nu+1)(N-\nu+2)(N-\nu+3)(N+1)(N+2)(N+3)}\gamma_3(N)g^\nu_{N+3}$$
$$- ((N-\nu)(N-\nu-1)(N-\nu-2)+N(N-1)(N-2))\gamma_3(N)g^\nu_N - (1/\tau^\nu_N)g^\nu_N. \tag{22}$$

The time constant

$$\tau^\nu_N = \left\{\frac{\nu^2}{4N}\left[W^+(N)+W^-(N)\right] + \nu^2 R_{00}(N) - \frac{i\nu\mu_N}{\hbar}\right\}^{-1}. \tag{23}$$

determines the time scale on which the non-diagonal matrix elements decay. We have assumed that $\nu \ll \bar{N}$, and therefore approximated

$$\frac{i}{\hbar}\left(E_0(N) - E_0(N-\nu)\right) \approx \frac{i\nu\mu_N}{\hbar}. \tag{24}$$

A. Stationary solution

First we neglect trap loss and solve Eq. (22) for the stationary case. All g^ν_N with $\nu \neq 0$ drop off exponentially in time, and we are left with the diagonal terms only. We use the detailed balance condition to calculate the stationary solution $\bar{g}^0_N$:

$$\frac{\bar{g}^0_{N+1}}{\bar{g}^0_N} = \frac{W^+(N)}{W^-(N+1)} \approx \mathrm{e}^{(\mu-\mu_N)/kT}, \tag{25}$$

and therefore

$$\bar{g}^0_N \propto \prod_{n=0}^{N} \mathrm{e}^{(\mu-\mu_n)/kT}. \tag{26}$$

Using the Thomas-Fermi approximation for the chemical potential of the condensate, and replacing the sum occurring in the exponent by an integral, we obtain

$$\bar{g}^0_N \propto \mathrm{e}^{N(\mu-5\mu_N/7)/kT}. \tag{27}$$

This particle distribution has one maximum determined by the condition $\mu_N = \mu$, as expected from thermodynamics. The position of the maximum differs from the mean number of particles in the condensate by an amount of order $1/\bar{N}$.

1. Linearized solution

In the case of $\bar{N} \gg 1$ we may linearize the solution Eq. (27) around the mean number of particles in the condensate $\bar{N}$. We approximate the distribution in Eq. (27) by a Gaussian

$$\bar{g}^0_N \approx \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(N-\bar{N})^2}{2\sigma^2}}. \tag{28}$$

The width σ of this Gaussian is given by

$$\sigma = \sqrt{\frac{5kT\bar{N}}{2\mu_{\bar{N}}}}. \tag{29}$$

In the Thomas-Fermi approximation σ scales with the mean number of condensate particles like $\bar{N}^{3/10}$. The difference between the linearized and nonlinearized solutions can therefore only be seen for very small condensates. Figure 3 shows a comparison between the Gaussian approximation and numerical solutions obtained from Eqs. (17) and (30). As expected both solutions agree very well with each other even for a mean occupation of the condensate of only about $\bar{N} \approx 500$. Note also that the same result for the variance may be obtained from statistical mechanics. Thus the restriction $\mu_{\bar{N}}/kT > 1$ is not necessary for this result to be valid.

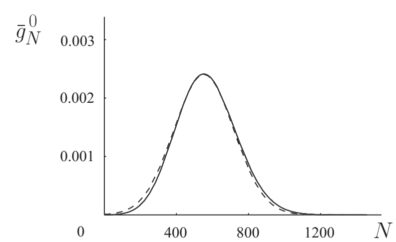

Fig. 3. Stationary particle distribution in the condensate. The trap loss is assumed to be zero. $\mu/kT = 0.05$ and $\mu_{N=1}/kT = 0.004$. The solid line represents the numerical stationary solution of the Fokker-Planck equation [Eq. (30)] and the detailed balance solution of the master equation [Eq. (17)]. The dashed line depicts the Gaussian approximation [Eq. (28)]. The trap frequencies are chosen to be $f_x = f_y = f_z/\sqrt{8} = 47$Hz. The calculation is done for Rubidium.

2. Inclusion of trap loss

In our model the bath of thermal atoms is not depleted by the interaction with the condensate. Experimentally this can be achieved by replenishing the reservoir by some mechanism, or by doing the experiment so quickly that the number of particles lost from the reservoir can be neglected. Furthermore, the calculations presented here for constant T and μ remain valid as long as the heat bath parameters μ and T change slowly compared to the time scale of the condensate dynamics. For the diagonal matrix elements g_N^0, we therefore obtain a stationary solution different from zero even if we include trap loss. Keeping only the leading order terms in N of Eq. (22), we immediately find the Fokker-Planck equation for g_N^0 to be

$$\dot{g}_N^0 = \frac{\partial}{\partial N}\{f_1(N)g_N^0\} + \frac{1}{2}\frac{\partial^2}{\partial N^2}\{f_2(N)g_N^0\} \tag{30}$$

where we have defined

$$\begin{aligned} f_1(N) = &-2NW^+(N) + 2N(W^-(N)+\gamma_1) \\ &+4N^2\gamma_2(N) + 6N^3\gamma_3(N), \end{aligned} \tag{31}$$

and

$$\begin{aligned} f_2(N) = &\,2NW^+(N) + 2N(W^-(N)+\gamma_1) \\ &+8N^2\gamma_2(N) + 18N^3\gamma_3(N). \end{aligned} \tag{32}$$

The Fokker-Planck equation is valid as long as $\bar{N} \gg 1$ and $\sigma \gg 1$.

We approximate the solution of this Fokker-Planck equation by a Gaussian, and obtain the following equation for the mean number of condensate particles $\bar{N}_{\rm loss}$:

$$\bar{f}_1 = 0. \tag{33}$$

The width of the Gaussian $\sigma_{\rm loss}$ is approximately given by

$$\sigma_{\rm loss} = \sqrt{\frac{\bar{f}_2}{2\partial_{\bar{N}_{\rm loss}}\bar{f}_1}}. \tag{34}$$

Using the assumption $\mu_{\bar{N}_{\rm loss}}/kT \ll 1$ Eq. (33) can be solved analytically by approximating

$$W^+(N) \approx W_a^+ = \frac{4m(akT)^2}{\pi\hbar^3}{\rm e}^{2\mu/kT}, \tag{35}$$

$$W^-(N) \approx W_a^-(N) = W_a^+\left(1 + \frac{\mu_N}{kT} + \frac{\mu_N^2}{2k^2T^2}\right){\rm e}^{-\mu/kT}. \tag{36}$$

For the mean number of particles in the condensate, we obtain

$$\bar{N}_{\rm loss} = \left(-\frac{\gamma_2(1)+\mu_1W_a^+{\rm e}^{-\mu/kT}/2}{3\gamma_3(1)+\mu_1^2W_a^+{\rm e}^{-\mu/kT}/2} + \sqrt{\left(\frac{\gamma_2(1)+\mu_1W_a^+{\rm e}^{-\mu/kT}/2}{3\gamma_3(1)+\mu_1^2W_a^+{\rm e}^{-\mu/kT}/2}\right)^2 - \frac{\gamma_1+W_a^+({\rm e}^{-\mu/kT}-1)}{3\gamma_3(1)+\mu_1^2W_a^+{\rm e}^{-\mu/kT}/2}}\right)^{5/2}. \tag{37}$$

Here $\mu_1 = \mu_{N=1}/kT$. Trap loss decreases the number of particles in the condensate. Also, the width of the particle distribution is decreased by the nonlinear loss. Both of these effects are well known in nonlinear optics [31]. Figure 4 shows the effect of trap loss on the mean number of particles in the condensate for a given T and μ for the parameters measured in Ref. [17]. The dominant contribution to the trap loss comes from three-particle loss, while one,- and two-particle losses are almost negligible. In the following sections we will omit the subscript loss since all the calculations will be done for finite trap loss.

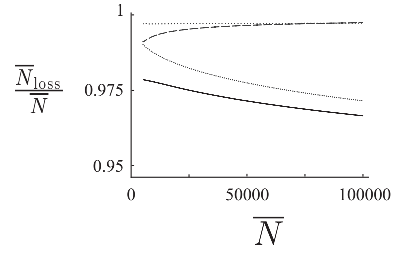

Fig. 4. Influence of trap loss on the mean number of particles in the condensate for rubidium. The trap frequencies are $f_x = f_y = f_z/\sqrt{8} = 47$Hz. The temperature of the thermal cloud is chosen $T = 0.5\mu K$. One,- two,- and three-particle losses are given by the dotted, dashed, and double dotted lines, respectively. The solid line accounts for all three kinds of loss. We have used $K_1 = 1/70s^{-1}$, $K_2 = 10^{-22}m^3/s$ and $K_3 = 6\cdot 10^{-42}m^6/s$ [17].

B. Nonstationary solutions

From Eq. (17) we find the evolution equation [32] for the mean number of condensed particles $\bar{N}$.

$$\begin{aligned} \dot{\bar{N}} = &\,2W^+(\bar{N})(\bar{N}+1) - 2W^-(\bar{N})\bar{N} \\ &-2\gamma_1\bar{N} - 4\gamma_2(\bar{N})\bar{N}^2 - 6\gamma_3(\bar{N})\bar{N}^3. \end{aligned} \tag{38}$$

By comparing Eq. (38) with the results obtained in Ref. [17] we find numerical values for the constants K_i. Equation (38) was investigated in Refs. [23,33] for the case $\gamma_i = 0$. Here we only want to show that trap loss

does not change the general behavior of the growth of the condensate. In Fig. 5 we show a comparison between the growth curve with and without trap loss. For the parameters chosen the time scale of condensate growth is influenced only slightly by trap loss.

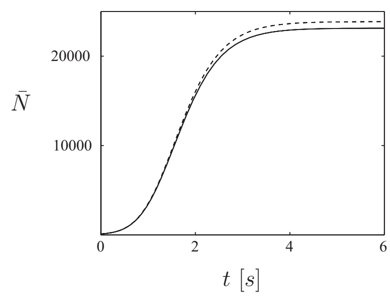

Fig. 5. Comparison between the growth of the condensate with trap loss (solid curve) and without trap loss (dashed curve). The parameters are chosen as $K_1 = 1/70\mathrm{s}^{-1}$, $K_2 = 10^{-22}\mathrm{m}^3/\mathrm{s}$, and $K_3 = 6 \cdot 10^{-42}\mathrm{m}^6/\mathrm{s}$ [17]. Parameters for the thermal particles are $T = 0.5\mu\mathrm{K}$ and $\mu = 3 \cdot 10^{-31}\mathrm{J}$. The trap frequencies are chosen to be $f_x = f_y = f_z/\sqrt{8} = 47\mathrm{Hz}$. The calculation is done for rubidium.

C. Correlation functions

In this section we calculate the intensity and amplitude correlation functions for a condensate in the stationary state found in Sec. IV A.

1. Ito equation

First we will show that if we restrict ourselves to situations where a large number of particles occupies the condensate and the density operator is almost diagonal in the basis $|N\rangle_N$, we can approximate Eq. (17) by a master equation of standard Lindblad form. To do so, we consider terms of the form

$$2C^\dagger\{F(\hat{N})\rho\}C - [CC^\dagger, \{F(\hat{N})\rho\}]_+, \tag{39}$$

define the operator $D = \sqrt{F(\tilde{N})}C$, and compare Eq. (39) with

$$2D^\dagger \rho D - [DD^\dagger, \rho]_+ \tag{40}$$

for a matrix element g_N^ν. Since we assume the trap loss to be a small effect compared to the interaction of the condensate with the thermal particles, the terms $C^\dagger C F(\hat{N})$ of the master equation (17) are of order $W^+\bar{N}$. The only exception are the terms involving R_{00} [34]. Therefore, we find for all the terms in our master equation that the difference of Eqs. (39) and (40) for a matrix element g_N^ν is of the order of $\bar{N}W^+(\bar{N})(\nu/\bar{N})^2$. We will linearize the equations and neglect all terms of order $W^+(\bar{N})/\bar{N}$. Therefore, we approximate the master equation (17) by replacing all the terms of the form Eq. (39) by expressions of the form of Eq. (40). This enables us to write down the Ito equation straightforwardly for the evolution of the system operators in the Heisenberg picture. Note that the noise terms appearing in the Ito equations do not have a direct physical interpretation. We only need them to have a mathematical equivalence between the solutions of the master equation and the solutions of the Ito equation. The Ito stochastic equation for an operator X in the Heisenberg picture reads

$$\begin{aligned}
dX = \Big\{ &\frac{i}{\hbar}[H_0, X] - [X, \sqrt{W^+}BB^\dagger\sqrt{W^+}]_+ + 2\sqrt{W^+}BXB^\dagger\sqrt{W^+} - [X, \sqrt{W^- + \gamma_1}B^\dagger B\sqrt{W^- + \gamma_1}]_+ + \\
&2\sqrt{W^- + \gamma_1}B^\dagger XB\sqrt{W^- + \gamma_1} - [X, \sqrt{R_{00}}(BB^\dagger)^2\sqrt{R_{00}}]_+ + 2\sqrt{R_{00}}BB^\dagger XBB^\dagger\sqrt{R_{00}} \\
&-[X, \sqrt{\gamma_2}B^\dagger B^\dagger BB\sqrt{\gamma_2}]_+ + 2\sqrt{\gamma_2}B^\dagger B^\dagger XBB\sqrt{\gamma_2} - [X, \sqrt{\gamma_3}B^\dagger B^\dagger B^\dagger BBB\sqrt{\gamma_3}]_+ + 2\sqrt{\gamma_3}B^\dagger B^\dagger B^\dagger XBBB\sqrt{\gamma_3}\Big\}dt \\
&-\Big([X, \sqrt{2W^+}B]dC_1 - [X, B^\dagger\sqrt{2W^+}]dC_1^\dagger\Big) - \Big([X, \sqrt{2(W^- + \gamma_1)}B^\dagger]dC_2 - [X, B\sqrt{2(W^- + \gamma_1)}]dC_2^\dagger\Big) \\
&-\Big([X, \sqrt{2R_{00}}BB^\dagger]dC_3 - [X, \sqrt{2R_{00}}BB^\dagger]dC_3^\dagger\Big) - \Big([X, \sqrt{2\gamma_2}B^\dagger B^\dagger]dC_4 - [X, BB\sqrt{2\gamma_2}]dC_4^\dagger\Big) \\
&-\Big([X, \sqrt{2\gamma_3}B^\dagger B^\dagger B^\dagger]dC_5 - [X, BBB\sqrt{2\gamma_3}]dC_5^\dagger\Big),
\end{aligned} \tag{41}$$

where dC_i are Ito noise increments. The only expectation values that are different from zero are

$$\langle dC_i(t)dC_i^\dagger(t)\rangle = dt. \tag{42}$$

Note that all the rates appearing in Eq. (41) depend on the number operator $\tilde{N}$ and, that therefore we use relations like [for example, for $W^-(\tilde{N})$]

$$BW^-(\tilde N) = W^-(\tilde N+1)B \approx \left(W^-(\tilde N) + \frac{dW^-(\tilde N)}{d\tilde N}\right) B = (W^- + \partial_{\tilde N} W^-)B \tag{43}$$

to calculate commutators between these rates and B.

2. Intensity fluctuations

We define the operator

$$\delta I_B = \frac{\tilde N - \langle \tilde N\rangle}{\sqrt{\langle \tilde N\rangle}}, \tag{44}$$

where we omit the time dependence whenever this can be done without causing confusion. For δI_B we obtain

$$d\delta I_B = -\gamma_I \delta I_B dt + dC_I, \tag{45}$$

where

$$\begin{aligned} dC_I = \Big(&\sqrt{2W^+}BdC_1 + B^\dagger\sqrt{2W^+}dC_1^\dagger + \\ &-\sqrt{2(W^-+\gamma_1)}B^\dagger dC_2 - B\sqrt{2(W^-+\gamma_1)}dC_2^\dagger \\ &-2\sqrt{2\gamma_2}B^\dagger B^\dagger dC_4 - 2BB\sqrt{2\gamma_2}dC_4^\dagger \\ &-3\sqrt{2\gamma_3}B^\dagger B^\dagger B^\dagger dC_5 - 3BBB\sqrt{2\gamma_3}dC_5^\dagger\Big)\frac{1}{\sqrt{\bar N}}. \end{aligned} \tag{46}$$

We expand around $\bar N$ and obtain

$$\gamma_I = \partial_{\bar N}\bar f_1 \approx 2\bar N\partial_{\bar N}\bar W^- + \frac{8}{5}\bar\gamma_2\bar N + \frac{24}{5}\bar\gamma_3\bar N^2. \tag{47}$$

which is now only a function of the expectation value of $\tilde N$. Keeping only these highest-order terms, we can solve Eq. (45) and obtain

$$\delta I_B(t) = \delta I_B(0)\mathrm{e}^{-\gamma_I t} + \int_0^t \mathrm{e}^{-\gamma_I(t-t')}\, dC_I(t'). \tag{48}$$

Using $\langle dC_I(t)dC_I^\dagger(t)\rangle = \bar f_2 dt/\bar N$ and considering only $t+s \gg \gamma_I^{-1}$ we obtain for the correlation function $\langle \tilde N(t), \tilde N(s)\rangle$

$$\langle N(t), N(s)\rangle = \frac{\bar f_2}{2\gamma_I}\mathrm{e}^{-\gamma_I|t-s|} = \sigma^2\mathrm{e}^{-\gamma_I|t-s|}. \tag{49}$$

Note that the operator δI_B is of order 1. For $\gamma_2 = \gamma_3 = 0$ we find for the Mandel Q parameter

$$Q \approx \frac{5kT}{2\mu_{\bar N}} - 1. \tag{50}$$

This means that as long as there is a significant thermal bath, Q will always be larger than 0 since in this case $\mu_{\bar N}/kT < 1$ holds. However, since the result for Q also follows from statistical mechanics the restriction $\mu_{\bar N}/kT < 1$ is not necessary for this result to be valid, and we obtain sub-Poissonian statistics for $\mu_{\bar N}/kT > 5/2$.

3. Amplitude fluctuations

We use the Ito equation introduced above to calculate the phase fluctuations. In particular we calculate the correlation function

$$\langle B^\dagger(t)B(s)\rangle. \tag{51}$$

First we simplify Eq. (41) for $X = B$ and find

$$dB = -\gamma_B B dt + dC_B, \tag{52}$$

where to leading order in $\bar N$

$$\gamma_B = \frac{i\mu_{\bar N}}{\hbar} + \frac{16}{25}\bar R_{00} + \sqrt{\bar N}\delta I_B\left(\frac{i\partial_{\bar N}\mu_{\bar N}}{\hbar} + \frac{\gamma_I}{2\bar N} - \frac{32\bar R_{00}}{125\bar N}\right). \tag{53}$$

The terms of order $\bar W^\pm/\bar N$ have been neglected. Expressions like Eq. (53) appear in optics in connection with the Kerr effect [35]. The noise term reads

$$\begin{aligned} dC_B = &\left(\sqrt{2W^+} + \frac{\partial_N W^+}{\sqrt{2W^+}}B^\dagger B\right)dC_1^\dagger - \frac{\partial_N W^+}{\sqrt{2W^+}}BBdC_1 + \left(-\sqrt{2(W^-+\gamma_1)} - \frac{\partial_N W^-}{\sqrt{2(W^-+\gamma_1)}}BB^\dagger\right)dC_2 \\ &+\frac{\partial_N W^-}{\sqrt{2(W^-+\gamma_1)}}BBdC_2^\dagger + \left(-\frac{\partial_N R_{00}}{\sqrt{2R_{00}}}B^\dagger B - \sqrt{2R_{00}}\right)B\left(dC_3 - dC_3^\dagger\right) + \left(-\frac{\partial_N\gamma_2}{\sqrt{2\gamma_2}}B^\dagger B^\dagger B - 2\sqrt{2\gamma_2}B^\dagger\right)dC_4 \\ &+\frac{\partial_N\gamma_2}{\sqrt{2\gamma_2}}BBBdC_4^\dagger + \left(-\frac{\partial_N\gamma_3}{\sqrt{2\gamma_3}}B^\dagger B^\dagger B^\dagger B - 3\sqrt{2\gamma_3}B^\dagger B^\dagger\right)dC_5 + \frac{\partial_N\gamma_3}{\sqrt{2\gamma_3}}BBBBdC_5^\dagger. \end{aligned} \tag{54}$$

We define the operator $\Phi = \frac{1}{\sqrt{B^\dagger B+1}}B$ and find the following equation for Φ

$$d\Phi = \left(-\gamma_B dt + dC_\phi - \frac{1}{2\sqrt{\bar N}}d\delta I_B\right)\Phi \tag{55}$$

Keeping only the leading terms, we obtain for dC_ϕ

$$dC_\phi = \left(-\frac{\partial_N R_{00}}{\sqrt{2R_{00}}} B^\dagger B - \sqrt{2R_{00}}\right)\left(dC_3 - dC_3^\dagger\right) \quad (56)$$

and find

$$dC_\phi(t) dC_\phi(t) = 2\left(-\frac{16}{25}\bar{R}_{00} + \sqrt{\bar{N}}\delta I_B \frac{32\bar{R}_{00}}{125\bar{N}}\right) dt. \quad (57)$$

Equation (55) can be treated as a c-number equation for Φ, since i times the noise term in this equation has the properties of a classical noise term. We define an operator $\Phi = \mathrm{e}^{i\phi}$ and obtain, for its increment,

$$d\phi = \left(-\frac{\mu_{\bar{N}}}{\hbar} - \frac{\partial_{\bar{N}}\mu_{\bar{N}}}{\hbar}\sqrt{\bar{N}}\delta I_B\right) dt - i dC_\phi \quad (58)$$

In this equation the intensity noise dC_I is independent of the phase noise dC_ϕ, so that the equation for ϕ is very similar to the equations appearing in the phase diffusion model for a laser with colored noise [36]. We may use the same method as used in quantum optics to calculate the correlation function $\langle B^\dagger(t)B(s)\rangle$, and obtain

$$\langle B^\dagger(t)B(s)\rangle = \bar{N}\mathrm{e}^{\left(i\frac{\mu_{\bar{N}}}{\hbar} - \frac{16}{25}R_{00}\right)(t-s)} \times \mathrm{e}^{\left(-\left(\frac{\sigma\partial_{\bar{N}}\mu_{\bar{N}}}{\gamma_I\hbar}\right)^2\left(\gamma_I(t-s) + \mathrm{e}^{-\gamma_I(t-s)} - 1\right)\right)} \quad (59)$$

for $t > s$ in the stationary state. In the range of validity of our approximations the result for this correlation function Eq. (59) agrees with the result obtained in QKIII [Eq. (184) of Ref. [3]]. If we were to keep the terms of $O(1/\sqrt{\bar{N}})$ in the noise terms the phase noise would no longer be independent of the intensity noise and the solution to Eq. (58) would not be so easy to find. The correlation function (59) depends only on the time difference $t-s$. The spectrum is found by a Fourier transformation in the time difference $t-s$

$$S(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\Delta t\, \mathrm{e}^{-i\omega\Delta t}\langle B^\dagger(t+\Delta t)B(t)\rangle. \quad (60)$$

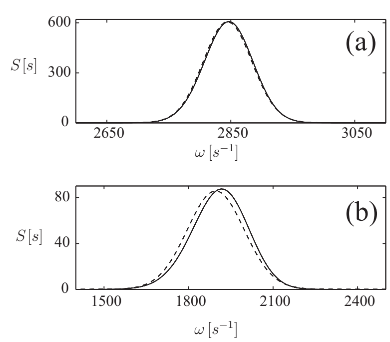

Fig. 6. Spectrum of the amplitude fluctuations $S(\omega)$ as defined in Eq. (60) against ω for rubidium. The trap frequencies are chosen to be $f_x = f_y = f_z/\sqrt{8} = 47$Hz. The numerical solutions are given by the solid lines, and the dashed lines represent the analytical results. In (a) the parameters of the thermal cloud are chosen to be $T = 0.25\mu K$ and $\mu = 3\cdot 10^{-31}J$. The mean number of particles therefore is $\bar{N} = 23800$. Plot (b) shows the spectrum for $T = 0.9\mu K$ and $\mu = 2\cdot 10^{-31}J$, and therefore a mean number of condensate particles $\bar{N} = 8640$.

Fig. 6 shows a comparison between the analytic result [Eq. (59)] and the direct numerical solution of Eq. (17). For a large number of condensate particles $\bar{N}$, the results agree very well with each other. In case of small particle numbers $\bar{N}$ the linearization used to obtain the analytic result shifts the curve compared to the numerical result. Even so, the shape of the solution is very well approximated by the analytic formula. The spectrum $S(\omega)$ is expected to be of Lorentzian shape around its maximum value. Further away from the center, the shape becomes Gaussian. However, for the parameters chosen in Fig. 6 the Gaussian part dominates.

V. CONCLUSIONS

In this paper we have calculated the correlation functions for amplitude (phase) and intensity fluctuations of a Bose condensate due to interactions with a heat and particle reservoir, representing uncondensed atoms at finite temperature. The present analysis is valid for a strongly condensed system confined in a trapping potential, ignoring contributions from quasiparticle excitations [3]. Finally, we point out that the present theory is readily adapted to a class of highly interesting problems, such as the study of decoherence in Josephson-like situations, where two trapped condensates are brought into contact and the quantum dynamics of the relative phase of the two condensates is observed [12,37].

ACKNOWLEDGMENTS

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APPENDIX A: STATIONARY SOLUTION FOR A CANONICAL BATH OF PARTICLES

We want to investigate the fluctuations in the number of particles of a Bose-Einstein condensate, assuming the system to be in the canonical ensemble. We will include interactions between the Bose particles in our analysis, and investigate their effects on the condensate fluctuations. On first glance one might expect that it would be satisfactory to account for the interaction effects by just including the mean field of the condensate (in the Thomas-Fermi approximation). However, in

this approach fluctuations in the size of the condensate lead to an unrealistically large shift of the energy levels, which prevents any condensation [25]. The inclusion of the mean field of the thermal density of particles reduces the shift of the energy levels. As long as fluctuations in the chemical potentials of the thermal cloud μ and the condensate μ_N are small compared to the energy gap between the condensate energy and the first excitation energy, fluctuations will not lead to degeneracy in the thermal cloud. The excitation energies therefore depend mainly on the total number of thermal particles M and the number of particles in the condensate N. We will assume that the eigenenergies of the excited levels depend on M and N but are independent of n_m, i.e., the microstate of the system. As shown in Ref. [38], the density of particlelike states is much larger than the density of quasiparticles. Thermodynamic quantities of the Bose gas are therefore mainly determined by the particlelike states, which allows us to use the Hartree-Fock approximation for describing the interactions in the thermal cloud.

1. Excited modes

We denote the energies and the wave functions of the excited states by ϵ_m and $\xi_m(\mathbf{x})$, respectively. The occupation of these levels is n_m. In the Hartree-Fock approximation [39], the effective potential for the thermal particles $V_{\text{eff}}(\mathbf{x})$ is given by

$$V_{\text{eff}}(\mathbf{x}) = V_T(\mathbf{x}) + 2uN\left|\xi_N(\mathbf{x})\right|^2 + 2u\tilde{n}(\mathbf{x}), \tag{A1}$$

with $\tilde{n}(\mathbf{x})$ the density of the noncondensed particles.

2. Weakly interacting Bose gas in the canonical ensemble

The density operator of a Bose gas in the canonical ensemble is given by

$$\rho_c = \frac{1}{Z_c}\mathrm{e}^{-\beta H}, \tag{A2}$$

where $\beta = 1/kT$, with T the temperature of the system. The partition function Z_c is given by $Z_c = \mathrm{tr}\left\{\mathrm{e}^{-\beta H}\right\}$. We want to investigate the properties of a Bose condensate in the canonical ensemble. The eigenstates $\{\xi_N(\mathbf{x}), \xi_m(\mathbf{x})\}$ and the corresponding eigenenergies $\{E_0, \epsilon_m\}$ depend on the number of condensate particles N and on the number of particles out of the condensate $M = \sum_m n_m$. The total number of particles N_{tot} is constant,

$$N_{\text{tot}} = N + M = \text{const.} \tag{A3}$$

The state of the system with N particles in the condensate and n_m particles in the levels ϵ_m is denoted by $|\mathbf{n}\rangle$ where $\mathbf{n} = \{N, n_1, ..., n_m, ...\}$. We can therefore write for the matrix elements of the density operator $p(\mathbf{n}) = \langle \mathbf{n}\,|\rho_c|\,\mathbf{n}\rangle$

$$p(\mathbf{n}) \propto \mathrm{e}^{-\beta E_0(N,M) - \beta\sum_m \epsilon_m(N,M)n_m} \tag{A4}$$

As can be seen from Eq. (A4), the condensate energy $E_0(N,M)$ and the excitation energies $\epsilon_m(N,M)$ are functions of the number of condensate particles N and the number of noncondensed particles M. In our calculations we will assume that the influence of the number of noncondensed particles M on the condensate energy is negligible, since the number of noncondensed particles that are inside the condensate region is much smaller than the number of condensed particles. Moreover, the width of the condensate particle distribution is only influenced by the change of the number of noncondensed particles in the condensate region due to fluctuations in M. The other interaction effects i.e., (i) the influence of the condensate mean field on the excited levels, (ii) the influence of the mean field of the thermal cloud on the excited levels, and (iii) the influence of the condensate mean field on the energy of the condensate, will be included in our calculations.

3. Particle number distribution of the condensate

Since we are only interested in the probability of finding N particles in the condensate we want to find

$$p(N,M) = \sum_{\{n_m\}} p(N, n_m) \tag{A5}$$

summed under the constrained $M = \sum_m n_m$. We can do this summation by introducing a contour integral writing

$$p(N,M) \propto \mathrm{e}^{-\beta E_0(N,M)} \times \frac{1}{2\pi i}\int_C \frac{dz}{z} z^{-M} \prod_m \sum_{n_m=0}^{\infty} \mathrm{e}^{-\beta\epsilon_m(N,M)n_m} z^{n_m}, \tag{A6}$$

and integrating using the method of steepest descents to obtain

$$p(N,M) \propto \mathrm{e}^{-\beta E_0(N,M) - (M+1)\ln(\bar{z}) - \sum_m \ln\left(1 - \bar{z}\mathrm{e}^{-\beta\epsilon_m(N,M)}\right)} \times \left(\frac{M+1}{\bar{z}^2} + \sum_m \frac{\mathrm{e}^{-2\beta\epsilon_m(N,M)}}{\left(1 - \bar{z}\mathrm{e}^{-\beta\epsilon_m(N,M)}\right)^2}\right)^{-1/2}. \tag{A7}$$

$\bar{z}$ depends on M and N, and is given by the solution of

$$M + 1 = \sum_m \frac{1}{\mathrm{e}^{\beta\epsilon_m(N,M)}\bar{z}^{-1} - 1}. \tag{A8}$$

By defining

$$F(N,M) = E_0(N,M) + \frac{M}{\beta}\ln(\bar{z}) + \frac{1}{\beta}\sum_m \ln\left(1 - \bar{z}\mathrm{e}^{-\beta\epsilon_m(N,M)}\right), \tag{A9}$$

$$C(N,M) = \left\{\ln\left(\partial_M \ln(\bar{z})\right) - \ln\left(\bar{z}\sum_m \frac{\beta\partial_M\epsilon_m(N,M)}{\mathrm{e}^{\beta\epsilon_m(N,M)} - \bar{z}} + \bar{z}^2 \sum_m \frac{\beta\partial_M\epsilon_m(N,M)}{\left(\mathrm{e}^{\beta\epsilon_m(N,M)} - \bar{z}\right)^2} + 1\right)\right\}\frac{1}{2}, \tag{A10}$$

and using Eq. (A8), we may rewrite Eq. (A7)

$$p(N,M) \propto \mathrm{e}^{-\beta F(N,M) + C(N,M)}. \tag{A11}$$

$C(N,M)$ is a small logarithmic correction to $\beta F(N,M)$ that we neglect in our further calculations. Consistent with the neglect of $C(N,M)$, we approximate $M+1$ by M in Eq. (A8) and use the equations

$$M = \sum_m \frac{1}{\mathrm{e}^{\beta\epsilon_m(N,M)}\bar{z}^{-1} - 1}, \tag{A12}$$

and

$$p(N,M) \propto \mathrm{e}^{-\beta F(N,M)}. \tag{A13}$$

$F(N,M)$ is the Helmholtz free energy of the system. The chemical potentials of the condensate and the thermal bath are therefore given by the partial derivatives of $F(N,M)$ with respect to N and M, respectively.

a. Stationary solution

The chemical potential of the condensate in the canonical ensemble $\mu_c(N,M)$ is given by

$$\mu_c(N,M) = \partial_N F(N,M) = \partial_N E_0(N,M) + \sum_m \frac{\bar{z}\partial_N\epsilon_m(N,M)}{\mathrm{e}^{\beta\epsilon_m} - \bar{z}}, \tag{A14}$$

and the chemical potential of the noncondensed particles in the canonical ensemble $\mu_v(N,M)$ reads

$$\mu_v(N,M) = \partial_M F(N,M) = \frac{\ln(\bar{z})}{\beta} + \partial_M E_0(N,M) + \sum_m \frac{\bar{z}\partial_M\epsilon_m(N,M)}{\mathrm{e}^{\beta\epsilon_m} - \bar{z}} \tag{A15}$$

If the total number of particles is fixed, the condensate particle distribution will reach its maximum value for $N = \bar{N}$ and $M = \bar{M} = N_{\mathrm{tot}} - \bar{N}$, where $\bar{N}$ can be found by solving

$$\mu_v(\bar{N}, N_{\mathrm{tot}} - \bar{N}) = \mu_c(\bar{N}, N_{\mathrm{tot}} - \bar{N}). \tag{A16}$$

The width of the stationary canonical distribution σ_{can} is given by

$$\sigma_{\mathrm{can}} = \left\{\beta\partial_{\bar{N}}\left(\mu_c(N, N_{\mathrm{tot}} - N) - \mu_v(N, N_{\mathrm{tot}} - N)\right)\right\}^{-1/2}. \tag{A17}$$

In the above equations (A14) and (A15), the terms including derivatives of $\epsilon_m(N,M)$ are corrections to the chemical potentials due to the shift of the excited energy levels with changing M and N. The term $\partial_M E_0(N,M)$ in Eq. (A15) is the correction to the chemical potential of the thermal bath due to changes in the condensate energy. By assuming $E_0(N,M) \approx E_0(N)$, we will neglect this term.

b. Method of solving for $p(N,M)$

For simplicity we will assume that the energy and wave function of the condensate are only functions of the number of condensed particles, and use the Thomas-Fermi expressions (11) and (12), respectively. The influence of the thermal particles on the energy and wave function of the condensate will become important at temperature close to T_c and may therefore only be neglected at very low temperatures [25].

We replace the sum in Eq. (A12) by an integral

$$\sum_m \to \int d\epsilon\, g(\epsilon), \tag{A18}$$

where $g(\epsilon)$ is the density of states and use the semiclassical expression

$$g(\epsilon) = \frac{1}{(2\pi\hbar)^3}\int d^3x \int d^3p\, \delta\left(\epsilon - \frac{\mathbf{p}^2}{2m} - V_{\mathrm{eff}}(\mathbf{x})\right), \tag{A19}$$

to obtain, for a spherically symmetric effective potential and given M and N,

$$M = 4\pi\int_0^\infty r^2 dr\, \tilde{n}(r) = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} 4\pi \int_0^\infty r^2 dr\, G_{3/2}\left(\frac{V_{\mathrm{eff}}(\mathbf{x})}{kT} - \ln(\bar{z})\right). \tag{A20}$$

Here $G_\sigma(x) = \sum_{n=1}^\infty n^{-\sigma}\mathrm{e}^{-nx}$ denotes the Bose functions. By solving this equation numerically we obtain $\bar{z}$ as well as the density of noncondensed particles $\tilde{n}(r)$. Given a fixed total number of particles N_{tot} and a temperature T of the system, we find the particle distribution function $p(N,M)$ from Eq. (A13) by replacing

$$-\sum_m \ln\left(1 - \bar{z}\mathrm{e}^{-\beta\epsilon_m(N,M)}\right) = \left(\frac{2mkT}{\pi\hbar^2}\right)^{3/2}\frac{\pi}{2}\int_0^\infty r^2 dr\, G_{5/2}\left(\frac{V_{\mathrm{eff}}(\mathbf{x})}{kT} - \ln(\bar{z})\right). \tag{A21}$$

in the expression Eq. (A9) for $F(N,M)$.

c. Solutions

We want to compare $p(N,M)$ with the solution for a grand canonical bath of thermal particles not influenced by mean-field effects as given in Eq. (28). To distinguish between the effect of choosing a different thermodynamic ensemble and the effect of including the change of the mean field due to condensate fluctuations, we will plot a third distribution obtained from the canonical ensemble with a fixed mean effective potential $\bar{V}_{\rm eff}(\mathbf{x})$ that is given by

$$\bar{V}_{\rm eff}(\mathbf{x}) = V_T(\mathbf{x}) + 2u\bar{N}\,|\xi_{\bar{N}}(\mathbf{x})|^2 + 2u\bar{n}(\mathbf{x}), \qquad \text{(A22)}$$

where $\bar{n}(\mathbf{x})$ is the thermal density obtained for $\bar{N}$ particles in the condensate, and $N_{\rm tot} - \bar{N}$ particles in the thermal cloud.

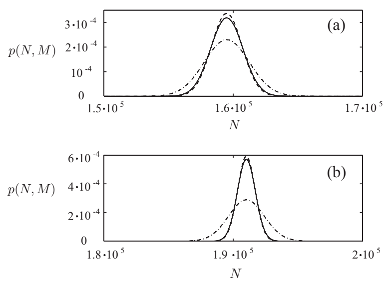

Fig. 7. Condensate particle distribution for (a) $T = 0.35\mu K$, $\bar{N} = 1.595 \cdot 10^5$, and $\bar{N}/N_{\rm tot} \approx 10\%$, and (b) $T = 0.2\mu K$, $\bar{N} = 1.910 \cdot 10^5$, and $\bar{N}/N_{\rm tot} \approx 33\%$. Both plots are for Rb and a trap frequency of $f_x = f_y = f_z = 66$Hz. The solid line shows the solution for the canonical ensemble including the mean-field effects, the dashed line is the canonical solution using the mean effective potential $\bar{V}_{\rm eff}(\mathbf{x})$, and the dash-dotted line is the grand canonical result.

In Fig. 7 , we plot the different particle distributions. For the three curves in each set the number of particles in the condensate $\bar{N}$ is the same. Since the inclusion of the mean-field effects shifts the chemical potential of the noncondensed atoms, the two canonical solutions are not plotted for the same total number of particles. If we were to plot the canonical solutions with and without inclusion of the mean-field effects at the same total number of particles $N_{\rm tot}$, their maxima would be shifted against each other. As can be seen from Fig. 7 , the stationary state of a Bose condensate in the canonical ensemble is different from the stationary state of a Bose condensate interacting with a bath of particles in the grand canonical ensemble. When the ratio $\bar{N}/N_{\rm tot} \approx 10\%$ this difference is not significant, but for larger condensates, i.e., $\bar{N}/N_{\rm tot} > 30\%$, the difference might become substantial.

As shown in this appendix, the model used to describe the thermal bath of atoms may have some influence on the particle fluctuations in the condensate. We also expect the intensity relaxation rate γ_I to be influenced by the particular model used for the noncondensed particles. However, a further more detailed analysis of such effects including a treatment of the excitation modes as given in Ref. [40] lies beyond the scope of this paper, and will be presented elsewhere [25,26].

APPENDIX B: CALCULATION OF $R_{00}(N)$

We show how the expressions (20) are derived from Eq. (141) in Ref. [3].

1. Simplifying the integral

R_{00} is defined by

$$R_{00}(N) = \frac{4u^2}{(2\pi)^5\hbar^2}\int d^3\mathbf{u}\int d^3\mathbf{K}_1\int d^3\mathbf{K}_2\int d^3\mathbf{k}\int d^3\mathbf{k}'\,\delta(\mathbf{K}_1-\mathbf{K}_2-\mathbf{k}+\mathbf{k}')F(\mathbf{K}_1,\mathbf{u})(1+F(\mathbf{K}_2,\mathbf{u}))$$
$$W_0(\mathbf{u},\mathbf{k})W_0(\mathbf{u},\mathbf{k}')\delta(\Delta\omega_{12}(\mathbf{u})), \qquad (1)$$

where

$$W_0(\mathbf{u},\mathbf{k}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{v}\,\xi_N^*(\mathbf{u}+\frac{\mathbf{v}}{2})\xi_N(\mathbf{u}-\frac{\mathbf{v}}{2})\mathrm{e}^{i\mathbf{k}\cdot\mathbf{v}}, \qquad (2)$$

and $\Delta\omega_{12}(\mathbf{u})$ accounts for energy conservation. We assume that within the range of the condensate $F(\mathbf{K},\mathbf{u})$ is constant, and that the factor $1+F(\mathbf{K}_2,\mathbf{u}) \approx 1$. Integrating over $\mathbf{K}_2$ and defining $\mathbf{Q} = \mathbf{k}-\mathbf{k}'$ yields

$$R_{00}(N) = \frac{4u^2}{(2\pi)^5\hbar^2}\frac{2m}{\hbar}\int d^3\mathbf{K_1}\int d^3\mathbf{Q}\,\delta(\mathbf{Q}\cdot(\mathbf{Q}+2\mathbf{K}_1))F(\mathbf{K}_1,0)$$
$$\int d^3\mathbf{k}'\int d^3\mathbf{u}\frac{1}{(2\pi)^6}\int d^3\mathbf{v}'\int d^3\mathbf{v}\,\xi_N^*(\mathbf{u}+\frac{\mathbf{v}}{2})\xi_N(\mathbf{u}-\frac{\mathbf{v}}{2})\xi_N^*(\mathbf{u}+\frac{\mathbf{v}'}{2})\xi_N(\mathbf{u}-\frac{\mathbf{v}'}{2})\mathrm{e}^{i\mathbf{k}'\cdot(\mathbf{v}+\mathbf{v}')+i\mathbf{Q}\cdot\mathbf{v}}. \qquad (3)$$

By integrating over $\mathbf{k}'$ and over $\mathbf{v}'$ and defining

$$G^2(\mathbf{Q}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{u} \int d^3\mathbf{v} \left|\xi_N(\mathbf{u} + \frac{\mathbf{v}}{2})\right|^2 \left|\xi_N(\mathbf{u} - \frac{\mathbf{v}}{2})\right|^2 \mathrm{e}^{i\mathbf{Q}\cdot\mathbf{v}} \tag{4}$$

we obtain

$$R_{00}(N) = \frac{4u^2}{(2\pi)^5\hbar^2}\frac{2m}{\hbar} \int d^3\mathbf{K}_1 \int d^3\mathbf{Q}\, \delta(\mathbf{Q}\cdot(\mathbf{Q}+2\mathbf{K}_1)) F(\mathbf{K}_1, 0) G^2(\mathbf{Q}). \tag{5}$$

Now we approximate $\mathbf{Q} + 2\mathbf{K}_1 \approx 2\mathbf{K}_1$, since $G^2(\mathbf{Q})$ is sharply peaked around $\mathbf{Q} = \mathbf{0}$ compared to the width of $F(\mathbf{K}_1, 0)$. Replacing the notation $\mathbf{K}_1$ by $\mathbf{K}$, choosing the K_z axis in the direction of $\mathbf{Q}$, we obtain

$$R_{00}(N) = \frac{4u^2 m}{(2\pi)^5\hbar^3} \int d^3\mathbf{Q} \int dK_x \int dK_y \int dK_z \delta(QK_z) \mathrm{e}^{(-\frac{\hbar^2(K_x^2+K_y^2+K_z^2)}{2m}+\mu)/kT} G^2(\mathbf{Q}). \tag{6}$$

Here Q denotes the modulus of $\mathbf{Q}$. Then we do the integration over $\mathbf{K}$ to obtain

$$R_{00}(N) = \frac{8u^2\pi m^2 kT}{(2\pi)^5\hbar^5}\mathrm{e}^{\mu/kT} \int d^3\mathbf{Q}\, \frac{G^2(\mathbf{Q})}{Q}. \tag{7}$$

So far we have not assumed a particular form of the trapping potential or the condensate wave function. These properties are contained in the function $G(\mathbf{Q})$.

B. Calculating the function $G(\mathbf{Q})$ for the harmonic oscillator

The trapping potential is of the form $V_T(\mathbf{x}) = ax^2 + by^2 + cz^2$ with $a = m\omega_x^2/2$, $b = m\omega_y^2/2$ and $c = m\omega_z^2/2$. We change the variables by defining $\mathbf{x} = \mathbf{u} + \frac{\mathbf{v}}{2}$ and $\mathbf{y} = \mathbf{u} - \frac{\mathbf{v}}{2}$ and get

$$G^2(\mathbf{Q}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{x} \int d^3\mathbf{y}\ |\xi_N(\mathbf{x})|^2 |\xi_N(\mathbf{y})|^2 \mathrm{e}^{i\mathbf{Q}\cdot(\mathbf{x}-\mathbf{y})}, \tag{8}$$

and therefore

$$G(\mathbf{Q}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{x}\ |\xi_N(\mathbf{x})|^2 \mathrm{e}^{i\mathbf{Q}\cdot\mathbf{x}}. \tag{9}$$

Using the Thomas-Fermi approximation for the condensate wave function, defining $Q'_x = Q_x/\sqrt{a}$, $Q'_y = Q_y/\sqrt{b}$ and $Q'_z = Q_z/\sqrt{c}$, and changing the variables $x' = x\sqrt{a}$, $y' = y\sqrt{b}$ and $z' = z\sqrt{c}$, simplifies the integral to

$$G(\mathbf{Q}) = \frac{1}{(2\pi)^{3/2}\sqrt{abc}Nu} \int_{\mathbf{x}'^2<\mu_N} d^3\mathbf{x}'(\mu_N - \mathbf{x}'^2)\mathrm{e}^{i\mathbf{Q}'\cdot\mathbf{x}'}. \tag{10}$$

Integrating in spherical coordinates we obtain

$$\begin{aligned} G(\mathbf{Q}) &= \frac{2}{\sqrt{2\pi abc}NuQ'} \int_0^{\sqrt{\mu_N}} dr\,(r\mu_N - r^3)\sin(Q'r) \\ &= \frac{4}{\sqrt{abc}NuQ'^5}\left[(3-\mu_N Q'^2)\sin(Q'\sqrt{\mu_N}) - 3\sqrt{\mu_N}Q'\cos(Q'\sqrt{\mu_N})\right], \end{aligned} \tag{11}$$

where $Q' = \sqrt{Q_x'^2 + Q_y'^2 + Q_z'^2}$.

C. Integration over Q

Last we will now find an expression for the integral

$$\Omega = \int d^3\mathbf{Q}\frac{G^2(\mathbf{Q})}{Q}. \tag{12}$$

We make use of the fact that $G(\mathbf{Q})$ only depends on Q' by replacing $d^3\mathbf{Q} = \sqrt{abc}\ d^3\mathbf{Q}'$ using spherical coordinates, and change the variable $Q' \to \sqrt{\mu_N}Q' = T$. The modulus Q can be expressed in terms of $\mathbf{Q}'$ as $Q = Q'\sqrt{a\sin^2\theta\cos^2\phi + b\sin^2\theta\sin^2\phi + c\cos^2\theta}$. We obtain

$$\Omega = \frac{8\mu_N^4}{\pi\sqrt{abc}N^2u^2} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \frac{\sin(\theta)}{\sqrt{a\sin^2\theta\cos^2\phi + b\sin^2\theta\sin^2\phi + c\cos^2\theta}} \int_0^{\infty} dT \frac{1}{T^9}\left[(3-T^2)\sin(T) - 3T\cos(T)\right]^2. \tag{13}$$

Next we integrate over T, define $x = \cos\theta$, $\alpha^2 = c - a\cos^2\phi - b\sin^2\phi$, and $\beta^2 = (a\cos^2\phi + b\sin^2\phi)/\alpha^2$, and write

$$\Omega = \frac{\mu_N^4}{9\pi\sqrt{abc}N^2u^2} \int_0^{2\pi} d\phi \frac{1}{\alpha} \int_{-1}^{1} \frac{dx}{\sqrt{x^2+\beta^2}}$$

$$= \frac{2\mu_N^4}{9\pi\sqrt{abc}N^2u^2} \int_0^{2\pi} d\phi \frac{\operatorname{arcsinh}(\beta^{-1})}{\alpha}. \tag{14}$$

In the case $\alpha^2 < 0$ we take the modulus of α^2 and replace arcsinh $\to$ arcsin. To further evaluate this integral we now assume that $a = b$. Then $\alpha^2 = c - a$ and

$\beta^2 = a/(c-a)$. The integration over ϕ then yields a factor of 2π, and we obtain the expressions given in Eqs. (20a) and (20b). $R_{00}(N)$ therefore depends on the number of particles in the condensate as $N^{-2/5}$.

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